Cosmological Parameters

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This article briefly summarizes the increasingly precise observational estimates of the cosmological parameters. After three years on the stump, the ΛCDM model is still the leading candidate. Although the Universe is expanding, our picture of it is coming together.

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1 Cosmic Connections, Complementarity, Concordance and Consistency

If your model of the Universe is a turtle, you want to know how big the turtle is, how old the turtle is, where the turtle came from and, in some obscure animistic models, how fast the turtle is expanding. Cosmological parameters are the observable quantities that most cosmologists think are important. In the context of general relativity and the hot big bang model, cosmological parameters are the numbers that, when inserted into the Friedmann equation,

$$H^2 = H_o^2 \left[\Omega_\Lambda + \Omega_k \, a^{-2} + \Omega_{\rm m} \, a^{-3} + \Omega_{\rm rel} \, a^{-4} \right], \tag{1}$$

best describe our particular observable Universe. The expansion is parametrized by Hubble's constant, $H_o = \dot{a}/a$, where a is the scale factor of the Universe. Observational estimates of the parameters in this equation, H_o, Ω_Λ, Ω_k, $\Omega_{\rm m}$ and $\Omega_{\rm rel}$ (and their sub-components) have been derived from hundreds of observations and analyses (e.g. Fig. 1). Table 1 is my attempt to summarize this immense body of work.

Table 1: Cosmological Parameters (Background)

Parameter	Estimate	Sub-Components	References
cosmological constant[a]	$\Omega_\Lambda = 0.7 \pm 0.1$		[1] – [9]
matter[b]	$\Omega_{\rm m} = 0.3 \pm 0.1$		‘ ’ + [10]
cold dark matter		$\Omega_{\rm c} = 0.26 \pm 0.1$	‘ ’ + [11]
baryonic matter[c]		$\Omega_{\rm b} = 0.04 \pm 0.01$	[11]
relativistic component[d]	$0.01 \lesssim \Omega_{\rm rel} \lesssim 0.05$		[12] [13] [14]
neutrinos[e]		$0.01 \lesssim \Omega_\nu \lesssim 0.05$	‘ ’
photons[f]		$\Omega_\gamma = 4.8^{+1.3}_{-0.9} \times 10^{-5}$	[15]
Hubble's constant[g]	$h = 0.72 \pm 0.08$		[16]
age of Universe[h]	$t_{\rm o} = 13.4 \pm 1.6$ Gyr		[2][3][4][17]
geometry[i]	$\Omega_k = 0.00 \pm 0.06$		‘ ’ + [12][18][19]
equation of state[j]	$w = -1.0^{+0.4}$		[20]
deceleration parameter[k]	$q_o = -0.05 \pm 0.15$		[6]
CMB temperature	$T_{\rm CMB} = 2.725 \pm 0.001$ K		[15]

[a] $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{crit}}$, $\rho_{crit} = \frac{3\,H_o^2}{8\pi G}$, $\rho_\Lambda = \frac{\Lambda}{3\,H_o^2}$, [b] $\Omega_{\rm m} = \Omega_{\rm c} + \Omega_{\rm b}$, [c] $\Omega_{\rm b} h^2 = 0.020 \pm 0.002$ [11], [d] $\Omega_{\rm rel} = \Omega_\nu + \Omega_\gamma$, [e] $0.04\ eV < m_{\nu,\tau} < 4.4\ eV$ [12],[13], [f] $\Omega_\gamma = 2.47 \times 10^{-5}\ h^{-2}\ T_{2.725}^4$ [21], [g] $h = H_o/100\ km^{-1} s^{-1} Mpc^{-1}$, [h] $t_{\rm o} = h^{-1} f(\Omega_{\rm m}, \Omega_\Lambda)$, see Fig.2, [i] $\Omega_k = 1 - \Omega_{tot}$, $\Omega_{tot} = \Omega_\Lambda + \Omega_{\rm m} + \Omega_{\rm rel}$, $\Omega_k = 0$ (flat), > 0 (open), < 0 (closed), thus $\Omega_{tot} = 1.00 \pm 0.06$, [j] $p = w\rho$, [k] $q_o = \Omega_{\rm m} - \Omega_\Lambda/2$

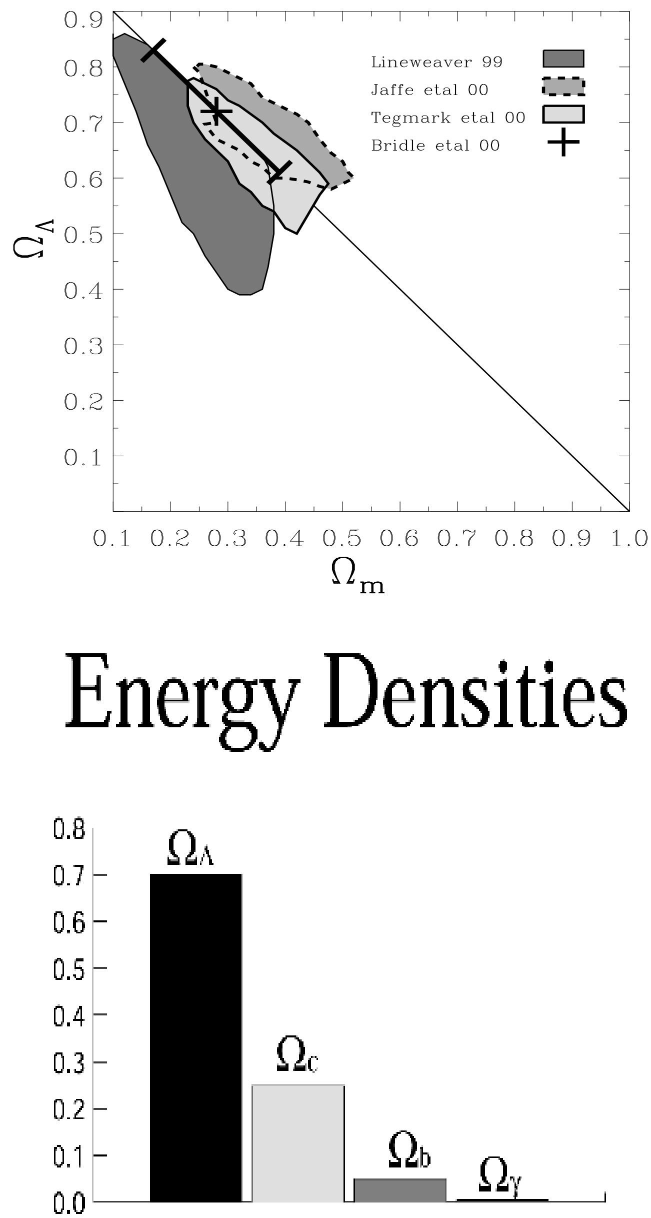

Figure 1: Various combinations of cosmic microwave background (CMB), supernovae and other observational constraints favor the region $(\Omega_\mathrm{m}, \Omega_\Lambda) \approx (0.3, 0.7)$ [1] – [7]. The most recent analyses, [8] [9], continue to favor this region. The composition of the Universe is thus, energy of the vacuum: 70%±10%, matter: 30%±10% (cold dark matter: 26%±10%, normal baryonic matter: 4% ± 1%), with negligible energy density from photons. The neutrino energy density is poorly constrained and may be as large as the baryonic energy density. About 13% of the matter in the Universe is baryonic ($\frac{\Omega_\mathrm{b}}{\Omega_\mathrm{m}} = \frac{0.04}{0.3} = 0.13$). The baryons can be further divided into 3% warm invisible gas, 0.5% optically visible stars and 0.5% hot gas visible in the x-rays [22]. The Ω_m in the top plot is equal to the sum of the Ω_c and Ω_b in the lower plot.

Various methods to extract cosmological parameters from cosmic microwave background (CMB) and non-CMB observations are forming an ever-tightening network of interlocking constraints. CMB observations tightly constrain Ω_k, while type Ia supernovae observations tightly constrain q_o. Since lines of contant Ω_k and constant q_o are nearly orthogonal in the $\Omega_{\rm m} - \Omega_\Lambda$ plane, combining these measurements optimally constrains our Universe to a small region (Fig. 1). Four years ago when Ω_Λ was assumed to be zero, the critical density, $\Omega_{crit} = \frac{3H_o^2}{8\pi G}$ *was* critical – it determined the fate of the universe – whether it would expand forever or recollapse. Currently, the notion of critical density has lost much of its importance. That role has been usurped by Ω_Λ; if $\Omega_\Lambda > 0$ the universe will expand forever.

The upper limit on the energy density of neutrinos comes from the shape of the small scale power spectrum. If neutrinos make a significant contribution to the density, they suppress the growth of small scale structure by free-streaming out of over-densities. The CMB power spectrum is not sensitive to such suppression and is not a good way to constrain Ω_ν. Hubble scholars used to be irreconcilably divided into camps described by a bimodal distribution peaking at $H_o = 50$ and $H_o = 90$. These peaks seem to have merged into a more agreeable Gaussian distribution peaking between 65 and 80 with error bars from hostile groups now overlapping.

The parameters in Table 1 are not independent of each other. The elongated contours in the top plot of Fig. 1 is one example of correlation. Another example is the age of the Universe, $t_{\rm o} = h^{-1} f(\Omega_{\rm m}, \Omega_\Lambda)$. Estimates of $h, \Omega_\Lambda, \Omega_{\rm m}$ can be inserted into Eq. 1. Integration then yields the age of the Universe ($\Omega_{\rm rel}$ is negligible and $\Omega_k = 1 - \Omega_{\rm m} - \Omega_\Lambda \approx 0$). If the Universe is to make sense, independent determinations of Ω_Λ, $\Omega_{\rm m}$ and h and the minimum age of the Universe must be consistent with each other. This is now the case (Fig. 2). Presumably we live in a Universe which corresponds to a single point in multidimensional parameter space. Estimates of h from HST Cepheids and the CMB must overlap. Deuterium and CMB determinations of $\Omega_{\rm b} h^2$ should be consistent. Regions of the $\Omega_{\rm m} - \Omega_\Lambda$ plane favored by supernovae and CMB must overlap with each other and with other independent constraints. This is the case [2].

The proportionality constant w in the equation of state, $p = w\rho$, is important in deciding whether the generalization of Ω_Λ into a time varying Ω_Λ (i.e. quintessence) is necessary. So far the observations seem to be favoring the simplest case $w = -1$, (i.e. pure cosmological constant) and do not call for this generalization.

Just as h describes the first derivative of the scale factor, the deceleration parameter q_o describes the second derivative. The redshifts and apparent magnitudes of type Ia supernovae have been used to find that $q_o \lesssim 0$ – the expansion of the Universe is accelerating. The geometry of the Universe does not seem to be like the surface of a ball ($\Omega_k < 0$) nor like a saddle ($\Omega_k > 0$) but seems to be flat ($\Omega_k \approx 0$) to the precision of our current observations.

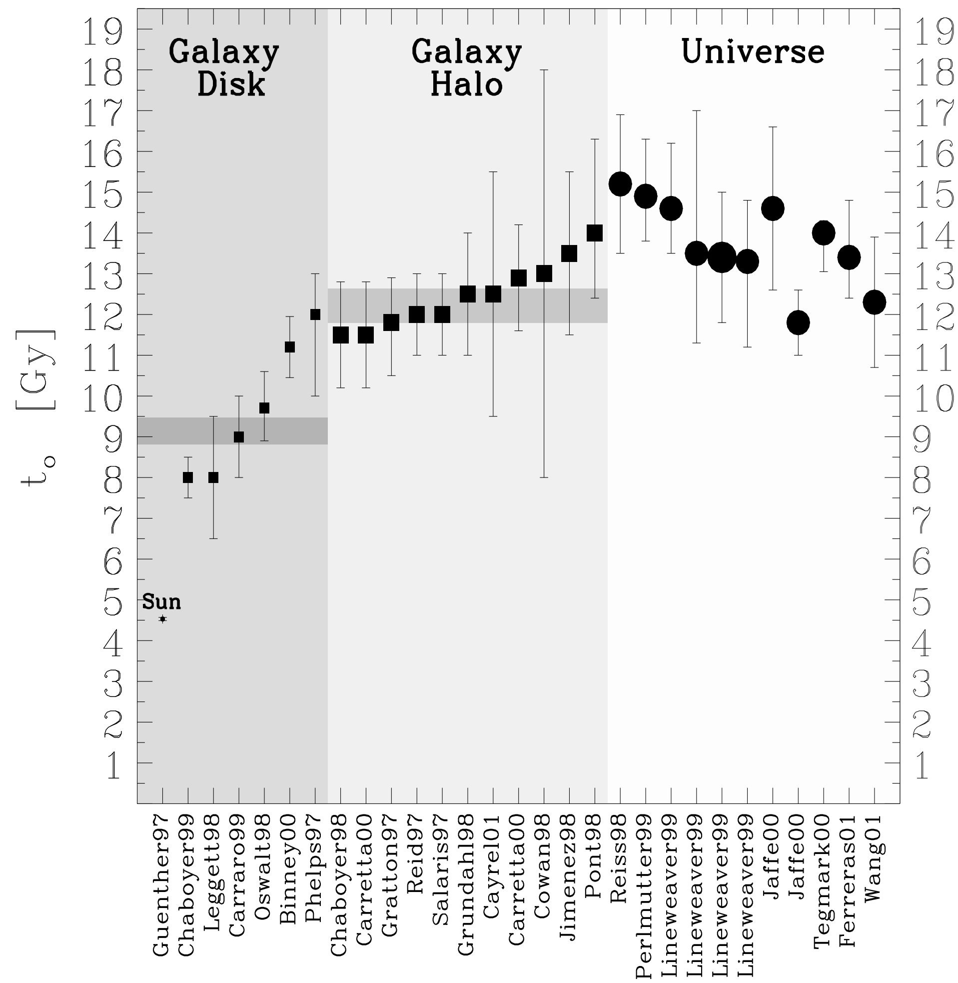

Figure 2: Estimates of the age of the Universe based on Eq. 1 are plotted on the far right ('Universe') and can be compared to lower limits on the age of the Universe from age estimates of the halo and disk of our Milky Way Galaxy. Many different techniques, data sets and analysis methods were used to obtain these estimates. Figure modified from [2].

2 Background and the Bumps on it

Equation 1 is our hot big bang description of the unperturbed GR-based Friedmann-Robertson-Walker Universe. There are no bumps in it, no over-densities, no inhomo-

geneities, no anisotropies and no structure. The parameters in it are the background parameters. It describes the evolution of a perfectly homogeneous universe [24].

However, bumps are important [25]. If there had been no bumps in the CMB thirteen billion years ago, no structure would exist today (Fig. 3). The density bumps seen as the hot and cold spots in the CMB map have grown into gravitationally enhanced light-emitting over-densities known as galaxies. Their gravitational growth depends on the cosmological parameters – much as tree growth depends on soil quality (see [26] for the equations of evolution of the bumps). Specifically, matching the power spectrum of the CMB (the C_ℓ which sample the $z \sim 1000$ universe) to the power spectrum of local galaxies ($P(k)$ which sample the $z \sim 0$ universe) can be used to constrain cosmological parameters. We measure the bumps and from them we infer the background.

3 GUTs, TOEs, Branes, Quintessence and Ekpyrotic Cycles

Many aspects of our Universe are more fundamental, but harder to estimate, than the cosmological parameters listed above. For example, the number of dimensions of our Universe is a useful parameter that would help us extend the frontiers of current research and smooth the inevitable transition from classical to quantum cosmology. The background and the bump parameters discussed in the previous sections were classical parameters.

Models of inflation usually consist of choosing a form for the potential. Estimates of the slope of the CMB power spectrum n_s and its derivative $\frac{dn_s}{dk}$ [27] may soon begin to constrain these potentials. Inflation solves the origin of structure problem with quantum fluctuations, and is just the beginning of quantum contributions to cosmology.

ΛCDM is an observational result that has yet to be theoretically confirmed. From a quantum field theoretic point of view $\Omega_\Lambda \sim 0.7$ presents a huge problem. It is a quantum term in a classical equation. But the last time such a quantum term appeared in a classical equation, Hawking radiation was discovered. A similar revelation may be in the offing. The Friedmann equation will eventually be seen as a low energy approximation to a more complete quantum model in much the same way that $\frac{1}{2}mv^2$ is a low energy approximation to pc.

Quantum cosmology is opening up many new doors. Varying coupling constants are expected at high energy [28] and c variation, G variation, α variation, and Ω_Λ variation (quintessence) are being discussed. We may be in an ekpyrotic universe or a cyclic one [29]. The topology of the Universe is also not without interest [30]. Just as we were getting precise estimates of the parameters of classical cosmology, whole new sets of quantum cosmological parameters are being proposed (see Liddle, these proceedings, for a discussion of inflation and some of the new cosmological ideas).

But there is hope. For example, inflationary models and the new ekpyrotic models make different predictions about the slope and amplitude of the tensor mode (n_T and A_T)

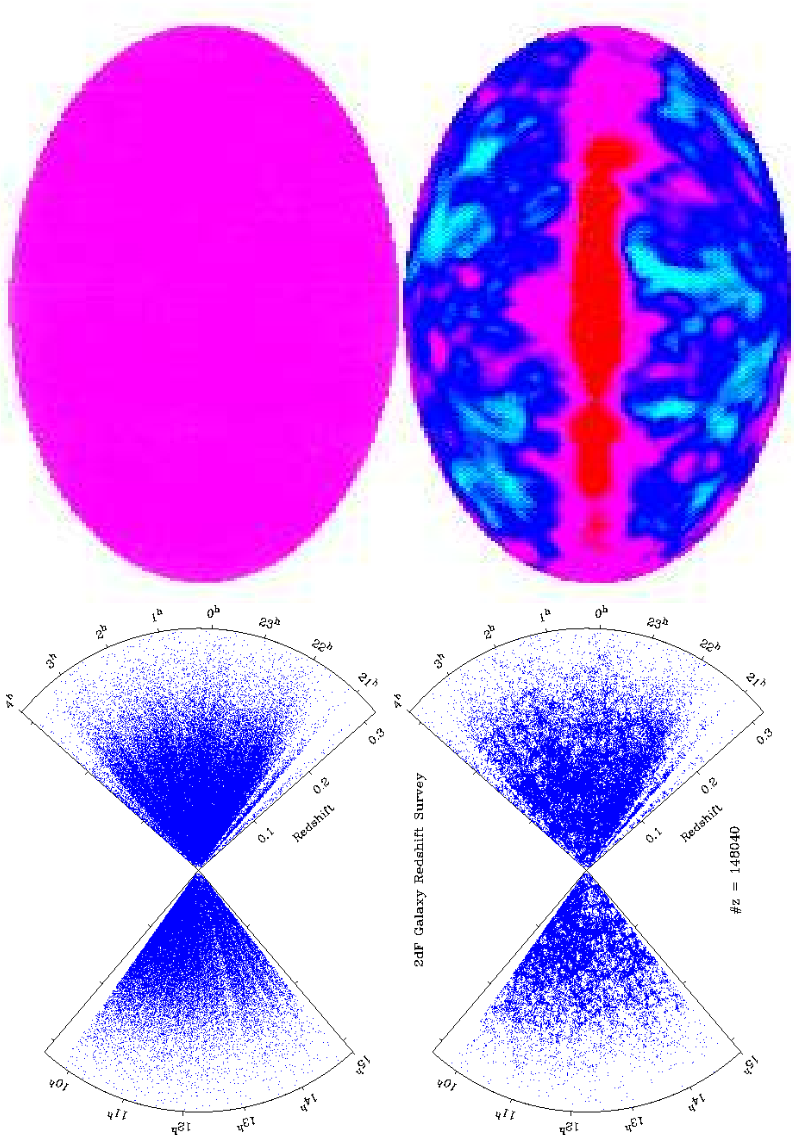

Figure 3: Background and the bumps in it. If there were no structure in the Universe then a full-sky microwave map would look like the map on the upper left – perfectly smooth and isotropic. The map in the upper right is the COBE-DMR map of the CMB at 53 GHz. On a scale of ± 150 μK around the mean temperature 2.725 K, the $\sim 30\mu$K ($\delta T/T \sim 10^{-5}$) bumps stand out. The vertical band is our Milky Way. These observable temperature bumps indicate the presence of density bumps: $\frac{\delta\rho}{\rho} = \frac{1}{3}\frac{\delta T}{T}$. If galaxies were distributed randomly in the Universe with no large scale structure, the 2dF galaxy redshift survey of the local universe would have produced a map like the one in the lower left. The map it did produce (lower right) shows galaxies clumped into clusters radially smeared by the fingers of God, with empty voids surrounded by great walls of galaxies.

contribution to the CMB power spectrum. Measurements of CMB polarization over the next few years will add more diagnostic power to CMB parameter estimation and may be able to usefully constrain n_T and A_T and distinguish these two models.

I have presented my version of the current best-fit cosmological parameters. Other versions can be found at [22][21][23] and Turner (this volume). By estimating cosmological parameters with precision, cosmology has learned to stick its neck out – it has become a real science, error bars and all. The ΛCDM model is still the leading candidate. Our classical picture of the Universe is falling into place even as Ω_Λ and the new mysteries of quantum cosmology enlarge the darkness, keeping the Universe safe for theoretical rogues and their poorly constrained speculations.

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